TITOLO TESI

Introduction

Long-term memory, or long-range dependence or long-term persistence, is a property of many natural phenomena, including climate patterns, earthquakes, and financial markets.

This property refers to the fact that the behavior of a system is dependent on its past behavior over a long period of time. In other words, events that occur far in the past can have a significant impact on the behavior of the system in the present.

In the field of time series analysis, LTM is an important area of research. One of the first contributions to this field was made by Edward Hurst, who studied the flow of the Nile River and developed a method to calculate the Hurst exponent a parameter that measures the degree of long-term memory in a time series.

Since Hurst's contribution, there has been much debate and research on the study of long-term memory in time series analysis, with applications in many fields.

In finance, the study of long-term memory is particularly relevant in understanding the behavior of financial markets, which exhibit complex dynamics and are affected by a wide range of factors, including economic indicators, geopolitical events, and investor sentiment.

In my thesis, the focus is finance and the use of Hurst exponent to analyze the behavior of S&P 500, the index that links the 500 large-cap companies listed on the New York Stock Exchange and NASDAQ, to understand a possible use of long-term memory in investment strategy.

# Time Series

In scientific analysis, measuring phenomena accurately is crucial to understanding how to evolve over time.

Time series data is a powerful tool, as it allows us to observe the evolution of a particular variable and describe it into trends and patterns that emerge over time.

This information can then be useful to forecast, that is one of the possible purposes.

One of the key advantages of time series analysis is that it allows us to detect and quantify dependencies between different observations.

This information can be used to make predictions about future movements, based on past behavior.

To conduct effective time series analysis, it's important to ensure that the data is accurate and consistent.

The starting hypothesis is that we want as much observations as possible to reduce the impact of noise and other random factors.

The frequency at which we collect data can also be an important consideration (we can use hour, daily, monthly, etc..), as it can affect the level of detail and resolution we can achieve in our analysis and possibility to erase outliers.

Time series data can be either continuous or discrete, depending on the nature of the observations.

A discrete time series is one in which observations are taken only at specific times, while a continuous time series involves observations that are made continuously over time. In both cases, the term "continuous" or "discrete" is used regardless of whether the measured variable can take on continuous or discrete values.

Graphical representation

Once we have collected and organized our time series data, we can represent it graphically using a variety of techniques. One of the most common approaches is to create a line plot with time on the x-axis and the observed variable on the y-axis. This allows us to visualize trends and patterns in the data over time, which can be a powerful way to communicate our findings to others.

By collecting accurate and consistent data and analyzing it effectively, we can gain insights into trends and patterns that can help us predict future behavior and make informed decisions in a variety of contexts.

Immagine che contiene testo, linea, Diagramma, schermata

Descrizione generata automaticamente

Just to make an example above we have a line plot that describes the time series of closing prices of Microsoft stock over time.

Different patterns and statistics

A preliminary analysis we can perform on a time series is the visual one because just look on the graph we can understand different patterns that could be:

* Horizontal, stationary series.
* Trend: a long-term component that represents the growth or decline in the time series over an extended time.
* Seasonal: a pattern that repeats itself year after year.
* Cyclical: component is the wavelike fluctuation around the trend.

Going on with the analysis of data there are some statistical measurements that could be performed on the dataset, here the most common:

1. Average: The average, or mean, is a measure of central tendency. It is calculated by adding up all the values in the time series and dividing by the number of observations. The average gives us a sense of the typical value of the time series and can be useful in comparing different time series.
2. Variance: the variance is a measure of the spread or variability of the time series. It is calculated by taking the average of the squared differences between each observation and the mean. A high variance indicates that the data points are spread out over a range of values, while a low variance indicates that the data are close the mean.
3. Standard deviation: is the square root of the variance and is often used as a more intuitive measure of variability. It tells us how much the observations deviate, on average, from the mean of the time series. A high standard deviation indicates that the data points are more spread out, while a low standard deviation indicates that the data points are more tightly clustered around the mean.
4. Autocovariance: is a measure of the linear relationship between two observations in a time series that are separated by a specific lag or time interval. It is calculated by taking the average of the product of the deviations from the mean at two time points, with the two time points being separated by the specified lag. The autocovariance can help identify patterns in the time series that are not captured by the mean, such as periodic fluctuations or trends.
5. Autocorrelation: is a measure of the linear relationship between two observations in a time series that are separated by a specific lag or time interval, after controlling for the effect of the mean. It is calculated by dividing the autocovariance at the specified lag by the variance of the time series. The autocorrelation can be used to identify patterns in the time series that are related to its own past values, such as seasonality or cyclicality.

A deeper analysis can evaluate some statistical properties of the data and there are some measures:

1. Skewness: measure of the asymmetry in data distribution. Positive skewness indicates that the distribution has a long tail on the right and negative skewness indicates a long tail on the left.
2. Kurtosis: measure of the "peakedness" of the distribution of the data. High value indicates that the distribution has a sharp peak and heavy tails, while low value indicates a more flat-topped distribution.
3. Cross-correlation: measure of the correlation between two time series at different lags. Cross-correlation can reveal the degree of dependence between two variables and the time lag at which they are most closely related.
4. Spectral analysis: A technique used to decompose a time series into its frequency components. This can reveal the dominant frequencies in the data and help identify seasonal or cyclical patterns that may not be immediately apparent.
5. Wavelet analysis: A method for analyzing signals in both the time and frequency domains. Wavelet analysis can identify changes in the frequency content of a time series over time, which can be useful for identifying abrupt changes in trend or seasonality.

Analysts to understand dynamic of time series uses a combination of technique. Ones they studied the data they go throw the forecast analysis.

Time series in finance

In finance, the use of time series analysis is critical because it allows analysts to examine how different variables change over time, understand their behavior, know their trend and the most important make forecast.

An example is the study of the Business Cycle, that is the cyclical nature of economy: growth, contraction, recovery.

The variables involved are stock prices, interest rates, exchange rates, commodity prices, and economic indicators such as inflation and GDP.

To analyze time series data and make forecasts, economists and financial analysts construct statistical models such as ARIMA, VAR, and GARCH. These models use historical data to identify patterns and relationships between variables, which are then used to make predictions about future movements in the markets. Some analysts use a combination of these models to achieve the highest possible accuracy in their forecasts.

In addition to forecasting, time series analysis can also be used to identify anomalies or outliers in the data. For example, if a stock's price suddenly drops or rises sharply, this may be indicative of some underlying event that is affecting the market. By identifying these anomalies, analysts can gain insights into what may be driving market movements and adjust their strategies accordingly.

Overall, time series analysis is a powerful tool for understanding the behavior of financial markets and making informed investment decisions.

By examining how different variables change over time and identifying patterns and trends, analysts can gain a deeper understanding of the factors that drive market movements and make more accurate forecasts.

Long-term memory

Once [briefly](https://context.reverso.net/traduzione/inglese-italiano/briefly) explained the concept of time series, graph representation, associated statistics, basic analytic tools, and some application with a focus in finance my thesis is going on introducing the topic of Long-term memory with which we can go deeply through a crucial point of this work that is the underlying assumption to ensure that everything works well.

In literature Long-term memory or long-range dependance or long-term persistence is related to the property of data to be influenced from past observations and is referred to a strong autocorrelation.

Autocorrelation

To better understand the concept could be useful a focus on autocorrelation; before I gave only a general definition.

Autocorrelation in a time series refers to the degree of similarity between a given time series, and a lagged version of itself, over successive time intervals.

The purpose of this is measure the relation between past and present data.

Therefore, a time series autocorrelation attempts to measure the current values of a variable against the historical data of that variable. It ultimately plots one series over the other and determines the degree of similarity between the two.

For the sake of comparison, autocorrelation is essentially the exact same process that you would go through when calculating the correlation between two different sets of time series values on your own. The major difference here is that autocorrelation uses the same time series two times: once in its original values, and then again once with a lag, generally “k”.

Testing autocorrelation

The next step is about testing autocorrelation in a time series and the most common tools is the correlogram also known as ACF plot.

In our case of study, we use auto-correlogram in which we take the time series of referment (Yt), we take the lagged values of the time series of referment (Yt) over time, where the lag is denoted by k. For example, Yt-k represents the time series with lag equal to k.

On the x-axis, we plot the lag values k, and on the y-axis, we plot the corresponding autocorrelation values.

At lag 0, the autocorrelation is always 1, and the maximum correlation value for non-zero lags is 1 as well. To facilitate the analysis, we can also include confidence intervals on the plot.

(graphical example)

Taken all the necessary information we can carry on with the definition of long-term memory and go deeply into the topic.

Time series with high level of long-term persistence is characterized by the presence of autocorrelations that decay slowly over time, meaning that events that occur at distant points in time are still dependent on each other.

The path of autocorrelation is clean in case of presence of long-term memory is a power-law.

Here some examples of long-term memory:

However, many physical and biological systems show presence of long memory or trends in the time series. For example, the number of particles emitted by a radiation source in a unit time decrease over time as the source becomes weaker. The density of air because of gravity changes at a different altitude following a trend. The air temperature, rainfall, and the water flow of rivers in different geographic locations show a periodic trend because of seasonal changes. Even the occurrence earthquakes show some kind of trend in certain areas. (Citare paper)

Hurst exponent

Given that the concept of long-term memory is relevant to many phenomena, the new challenge is to develop accurate methods and models for measuring it, in order to effectively exploit this property.

The most common method for evaluating the presence of long-term memory is the “Hurst method”. To illustrate the importance of long-term memory, we can look at one of the most precious resources, water, in one of the world's most important rivers, the Nile, which gave rise to one of the most long-lived civilizations - ancient Egypt by Harold Edwin Hurst.

Hurst was a hydrologist who spent a lot of time in Egypt between 1906 and 1968 and the first task of him and his team was to study a way to control Nile’s water and use a holistic vision of the problem because until that moment the using of Aswan damming had given poor results to the problem of irregular flow.

Hurst give a first approach and open to the discussion about Long-term memory; he was the first who observed this phenomenon.

In his work he used the first method to calculate the persistence, the measure of long-term memory with the R/S analysis.

This was the first estimation of this measure and he discover what is called “Hurst exponent”.

Interpretation of Hurst exponent

After performing the analysis, the coefficient that explain the degree of persistence could be a number between 0 and 1; a Hurst exponent of 0.5 signifies a random walk, while a value greater than 0.5 indicates persistence, and a value less than 0.5 indicates anti-persistence.

We can describe every threshold.

1. 0,5 < H ≤ 1: Persistence refers to the tendency of a variable or process to continue in a certain direction or behavior over time. Is the case linked to the presence of Long-term memory and is the optimal result in our analysis because we can use this property to forecast.
2. H = 0,5: random walk a mathematical model that describes the behavior of a variable or process that has no memory of its past values and is subject to random fluctuations over time.
3. 0 ≤ H< 0,5: anti-persistence

Methods to calculate Hurst Exponent

The crucial part of the thesis is the methodological approach; we already talked about what is a time series, we exploit the concept of Long-term memory and the Hurst Coefficient as a measure of degree of persistence in time series.

Now we can go on with different models to manage data and calculate the Hurst parameter.

Rescaled range (R/S)

As written in the previous paragraph the first method to estimate the Hurst exponent was the R/S analysis which gave the name to the parameter.

Calculate the range of the time series:

1. Let X(t) be the time series of length N. Divide the time series into non-overlapping sub-series of length k, and calculate the range R(k) for each sub-series by taking the difference between the maximum and minimum values: R(k) = max(X(i)) - min(X(i)), where i = 1, 2, ..., N/k.
2. Calculate the rescaled range: Calculate the rescaled range S(k) for each sub-series of length k as:

S(k) = (1/k) \* sum(R(k)) / std(X), where std(X) is the sample standard deviation of the time series.

1. Estimate the Hurst parameter: Plot the logarithm of the rescaled range log(S(k)) versus the logarithm of the sub-series length log(k) and estimate the slope of the line using linear regression. The Hurst parameter can be estimated as the slope of the line.

Aggregate variance

Another possible calculation for the Hurst parameter is the aggregate variance method based on the self-similarity property of the process.

1. Split the time series into different scales: Divide the time series into different subsets of equal length, with each subset containing 2^k data points, where k is an integer.
2. Calculate the mean and variance of each subset: For each subset, calculate the mean and variance of the data points.
3. Calculate the aggregate variance of the subsets by adding up the variances of each subset.
4. Plot the log-log relationship between the aggregate variance and the subset size: The slope of the line obtained from the plot will be equal to 2H, where H is the Hurst parameter.
5. Estimate the Hurst parameter as H = slope/2.

Difference variance

Used with absolute variance method is useful to make a distinction between a non-stationary sequence and long-range dependance sequence.

To make this analysis:

1. Calculate the range of the time series: Let X(t) be the time series of length N. Calculate the range R(k) for each sub-series of length k by taking the difference between the maximum and minimum values: R(k) = max(X(i)) - min(X(i)), where i = 1, 2, ..., N/k.
2. Calculate the variance of the range: Calculate the variance of the range for each sub-series of length k: V(k) = var(R(k)), where var(.) denotes the sample variance.
3. Estimate the Hurst parameter: Plot the logarithm of the variance of the range log(V(k)) versus the logarithm of the sub-series length log(k) and estimate the slope of the line using linear regression. The Hurst parameter can be estimated as the slope of the line.

Absolute moment

The Absolute Moment Method is the third way for estimating the Hurst parameter, which is based on the scaling of the absolute moments of the time series from the work of Taqqu in 1995.

Here are the steps to estimate the Hurst parameter using this method:

1. Compute the mean of the time series: Calculate the mean of the time series, denoted by μ.
2. Compute the absolute deviations: Calculate the absolute deviations of the time series from the mean, denoted by |X\_t - μ|.
3. Calculate the q-th order absolute moments: Calculate the q-th order absolute moments of the time series as M(q) = (1/N) \* sum(|X\_t - μ|^q), where N is the length of the time series.
4. Estimate the Hurst parameter: Estimate the Hurst parameter as H = (log(M(q)) - log(M(q/2))) / log(2).

Periodogram method

Before explicating the steps for the estimation of “H” we need to talk about the periodogram; is a tool for analyzing the spectral properties of a time series. It is obtained by taking the Fourier transform of the autocovariance function of the series. The periodogram is a plot of the power spectrum of the time series, which shows how the energy of the series is distributed across different frequencies.

1. Calculate the power spectrum of the time series: Calculate the power spectrum of the time series using the fast Fourier transform (FFT) algorithm. The power spectrum is the squared magnitude of the FFT coefficients. Denote the power spectrum by P(f), where f is the frequency.
2. Calculate the average power at each frequency: Divide the power spectrum into non-overlapping frequency bands of width delta\_f. Calculate the average power in each frequency band as: S(k) = (1/N) \* sum(P(f)), where f is in the k-th frequency band, and N is the length of the time series.
3. Calculate the logarithm of the average power: Take the natural logarithm of the average power at each frequency band, denoted by log(S(k)).
4. Estimate the Hurst parameter: Plot the logarithm of the average power log(S(k)) versus the logarithm of the frequency log(k) and estimate the slope of the line using linear regression. The Hurst parameter can be estimated as the slope of the line.

Modified periodogram

Another formulation is using the modified periodogram method that aims to reduce the bias and variability by using a modified spectral estimator.

The modified periodogram is obtained by applying a smoothing window to the periodogram of the time series. The smoothing window reduces the high-frequency noise in the periodogram and improves the accuracy of the estimation.

1. Compute the periodogram of the time series:Let X(t) be the time series of length N. Compute the periodogram of X(t) using the following formula: I(w) = (1/N) \* |FFT(X(t))|^2, where FFT(X(t)) is the fast Fourier transform of X(t) and |.|^2 denotes the complex conjugate.
2. Compute the modified periodogram: Compute the modified periodogram J(w) by averaging the periodogram over adjacent frequency bands: J(w) = (1/K) \* sum(I(w')/w') for w' in [(w/K),(w\*(K-1)/K)], where K is the number of frequency bands.
3. Estimate the Hurst parameter: Compute the log-log plot of J(w) versus w. The Hurst parameter can be estimated as the slope of the line fitted to the log-log plot.

HIGUCHI

Proposed by Higuchi in 1988 and is based on the use of fractal dimension of the time series, used to quantify the scaling properties. Here are the steps to estimate the Hurst parameter:

1. Choose a value for the maximum scale factor k. This value determines the maximum length of the subseries that will be analyzed. A typical value for k is around 10% of the length of the time series.
2. For each value of k, divide the time series into non-overlapping subseries of length k. The number of subseries is equal to N(k) = ⌊(N - 1) / k⌋, where N is the length of the time series.
3. Compute the length of each subseries, L(m,k), where m is the subseries index and k is the scale factor. The length of each subseries is calculated as follows:

L(m,k) = 1 + (N - 1 - k \* m) / k

1. Compute the mean length of the subseries over all values of m for a given value k: < L(k) > = (1 / N(k)) \* sum( L(m,k))
2. For each value of k, compute the estimate of the Hurst exponent as follows: H(k) = log(< L(k) >) / log(k)
3. Compute the average of the H(k) values over all values of k to obtain the final estimate of the Hurst exponent: H = (1 / K) \* sum(H(k))

Residuals of Regression

This method which was used by Penget involves detrending the time series and calculating the fluctuation around the trend using the residuals of a linear regression model fitted to the detrended series. The step to make the Hurst estimation are:

1. Detrend the time series: Fit a linear regression model to the time series and subtract the predicted values from the observed values to obtain the residuals.
2. Divide the residuals into non-overlapping blocks of length k.
3. Calculate the standard deviation (SD) of the residuals within each block.
4. Calculate the average SD across all blocks for a given block size k.
5. Repeat steps 2-4 for different block sizes.
6. Plot the log of the block size against the log of the average SD to obtain a log-log plot.
7. Estimate the slope of the line using linear regression. The slope is an estimate of the Hurst exponent.

Long-term memory in finance and calculation

We already know that a lot of variables in finance could be registered with time series to describe trends and visualize it.

That said, clearly, all the rules and evidence work and so we can talk of long-term memory even in finance that enables the quantification of historical patterns and trends.

By employing various calculation methods, finance professionals can assess the degree of persistence in financial data and make more informed decisions. Understanding long-term memory has implications for market predictability, investment strategy development, risk management, and performance evaluation. As the field of finance evolves, ongoing research into long-term memory calculation will continue to enhance our understanding of financial markets and support more effective financial decision-making.

MARKET EFFICIENCY

One of application of Hurst coefficient in finance is the field of Market Efficiency aligned with the Efficient Market Hypothesis (EMH), proposed by Fama states that stock prices reflect all available information. According to the EMH, asset prices fully reflect all available information, and it is difficult for market participants to consistently generate excess returns by trading on that information.

EHM could be divided into three types: Weak EMH (related to historical information, semi-strong EMH (related to public information) and strong-form EMH (related to future information)

Research is more focused in weak EMH, because we have the support of time series.

There is evidence that Hurst exponent also can be used to rank the efficiency of markets *(Cajueiro and Tabak (2004, 2005))* “the higher the Hurst exponent is, the lower the efficiency of the market is”.

In this paper the aim of the authors is to study the efficiency of emerging equity markets. They use Hurst coefficient as an efficient measure to make the comparison between markets and the methodology consist in two types of Hurst, the original one with R/S analysis applied to log return time series calculated using closing prices and another methodology proposed in *Erramili A, Narayan O, Willinger W. IEEE Trans Neural Networks 1996;4:209* that consist in perform the classic R/S analysis to block of shuffled data casually picked in small size group in this case 10 and apply R/S analysis.

They collect closing prices daily closing prices for Argentina, Brazil, Chile, India, Indonesia, Malaysia, Mexico, the Philippines, South Korea, Taiwan, Thailand, Japan, and the US from Jen 1991 and Jen 2004.

The Hurst is calculated in this way:

1. Start with a time window of 1000 observations.
2. Calculate the Hurst exponent for the time window.
3. Roll the sample one point forward, eliminating the first observation and including the next one.
4. Calculate the Hurst exponent for the new time window.
5. Repeat steps 3 and 4 until you reach the end of the series.

This process is repeated thousands of times, which allows us to get a more accurate estimate of the Hurst exponent.

The authors use table to show the difference between the two different calculation of Hurst and for every method they use squared returns and absolute returns both used as proxy for volatility.

Both measures of volatility lead to the same qualitative results but with absolute returns the exponents are higher.

Immagine che contiene testo, ricevuta, schermata, numero

Descrizione generata automaticamente

Here the table to rank the two methodologies for hurst coefficient for absolute returns and squared returns and give an idea to the efficiency of the market.

The focus is on the fact that higher is Hurst lower is the efficiency of the market.

The conclusion besides the ones linked to the study that Asian county are more efficient than Latin American lead to a general conclusion about the use this information about the long-term measures is welcome in option market. This paper contributes to highlight the importance of using Hurst exponents instead of relying on single static measures of long memory dependence.

Furthermore, there is strong evidence of long memory in equity volatility.

This conclusion and the findings of inverse correlation between efficiency and Hurst represented a starting point in economic literature to consider Hurst Coefficient as an efficiency measure.

Another contribution comes from *Mynhardt, R. H., Plastun, A., & Makarenko, I.(2014). Behavior of financial markets efficiency during the financial market crisis: 2007 – 2009.*

The authors here use the Hurst exponent to measure the efficiency of financial markets during the financial crisis.

The idea is to use the fractal market hypothesis (FMH) opposed to the efficient market hypothesis (EMH). For fractal Hypothesis the starting point is represented by Mandelbrot and then Peters, by this theory financial markets are not random, but rather have a self-similar structure. This means that the patterns that we see in the market today are the same as the patterns that we saw in the past and will see in the future. The FMH has been supported by several studies that have shown that financial markets do indeed exhibit self-similar behavior. For example, one study found that the distribution of returns on the stock market is self-similar over time scales ranging from days to years.

Today FMH is considered as an extension of widely utilized EMH; is important to focus on this aspect because during period of crisis the leading theories as EMH was putted in discussion.

The aim of this work is to investigate the market efficiency during different phasis (period of crisis and normal period) in different county.

Authors divided the data from two big groups:

* Developed country: U.S., Japan, UK, EU
* Developing country: China, India, Brazil, Russia, and Ukraine

The dataset is constituted by national stock market, and the exchange rate of national currency.

To evaluate the efficiency the methodology operated is the hurst coefficient and the results are divided in two parts pre crisis from 1990 to 2007, and crisis period from 2007 to 2010.

The Hurst is the classical R/S analysis for pre-crisis period and the findings are represented in a table.

As usual the thresholds for Hurst are:

* 0 ≤ H < 0,5: data is fractal, FMH is confirmed, «heavy tails» of distribution, antipersistent series and negative correlation.
* H = 0,5: data is random, EMH is confirmed, movement of asset prices is an example of the random Brownian motion (Wiener process), time series are normally distributed, lack of correlation in changes in value of assets (memory of series), white noise of independent random process
* 0,5 < H ≤ 1: data is fractal, FMH is confirmed, «heavy tails» of distribution, persistent series, positive correlation within changes in the value of assets, black noise and trend is present in the market.

Immagine che contiene testo, numero, ricevuta, Carattere

Descrizione generata automaticamente

As can be shown the trend is evident the difference of Hurst coefficient in stock market between developed countries highlights coefficient near the 0,5 that mean presence of long-term persistence and efficiency of the market.

While developing countries shown presence of long-term persistence but lower efficiency of the market.

About foreign exchange market the trend is almost the same, but the only disclaimer can be done about the high volatility of this market for developing country.

The second half of the study is about the crisis period the methodology is dynamic Hurst exponent consisting in calculate the Hurst exponent for different data windows, found that a window size of 300 (close to one calendar year) is the most suitable. This is because for narrower windows, the volatility of the Hurst exponent increases dramatically, while for wider windows, the Hurst exponent is almost constant, the dynamics are not apparent.

The authors then calculate the Hurst exponent for different dates, by shifting forward the data window by 10.

This was done to obtain enough estimates to analyze the behavior of the Hurst exponent.

Just to understand, the Hurst exponent for the date 13.07.2007 was calculated on the data for the period from 21.04.2005 till 13.07.2007. The second value was then calculated for 27.07.2006 and characterized the market over the period 10.05.2005 till 27.07.2006, and so on.

Results are a variety of control points; this methodology is essential to track the changing in persistence of market over time and have a focus on any changes during the financial crisis.

The dataset starts from 2007 to consider the period of bubble inflation and the market overheating which represented the starting point of the crisis in late 2008, till the end of crisis in 2010.

Immagine che contiene testo, numero, parole crociate, ricevuta

Descrizione generata automaticamente

To better understand the moments during the crisis and investigate the persistence changing using Hurst, authors choose to explicit a Minimum and a maximum value of the coefficient. Is evident that during crisis the EMH didn’t perform. In conclusion, the authors suggest that the Hurst exponent can serve as a "fear index" in finance, reflecting market conditions, future direction, uncertainty (volatility), and investor sentiment. An increasing Hurst exponent indicates ongoing market inefficiencies, while a lower exponent suggests a more efficient market. Therefore, the methodology of the Hurst exponent can be a valuable tool in measuring market efficiency.

Some debate about market efficiency became consistent talking of behaviors of market in crisis period. Economist find that the Hurst exponents of most financial markets decreased during the crisis period, which suggests that market efficiency decreased during this time this thesis is supported by several studies as *Grech D., Pamula, G. (2008), “The local Hurst exponent of the financial time series in the vicinity of crashes on the Polish stock exchange market”, Physica A :Statistical Mechanics and its Applications, Vol. 387, pp. 4299-4308.*

This is because the crisis led to increased volatility and uncertainty in the financial markets, which made it more difficult to predict the future values of financial assets. However, the authors also find that the Hurst exponents of some financial markets increased during the crisis period, which suggests that market efficiency increased for these markets. This is because the crisis led to increased trading volume and liquidity in some financial markets, which made it easier to trade and price financial assets.

The authors then classify financial markets of different countries by the level of their efficiency. They find that financial markets of developed countries are more efficient than the developing ones. This is consistent with the findings of other studies, which have shown that developed markets are more liquid and have lower transaction costs than developing markets.

The authors conclude that the financial market crisis had a significant impact on the efficiency of financial markets. They suggest that further research is needed to understand the long-term effects of the crisis on market efficiency.

The paper also discusses several limitations of the study. One limitation is that the study only examines a relatively short period of time, so it is not clear whether the findings will be sustained in the long run. Another limitation is that the study only uses a single measure of market efficiency, so it is possible that other measures of market efficiency would yield different results.

The Hurst coefficient in the study of development stage of market and applicability of this measure to efficiency is investigated even in “*Long term memories of developed and emerging markets: using the scaling analysis to characterize their stage of development T. Di Matteo a,b, T. Aste b, M. M. Dacorogna c.”*

The purpose of the article is to relate the scaling exponent and the development stage of the market.

To do this the authors use:

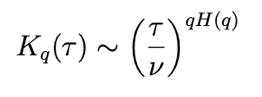
* 32 of the major indices of developed markets like the US or and emerging markets. Daily data from 1990 or 1993 to 2001.
* 29 daily spot rates of major currencies against the U.S. dollar. From 1990 to 2001 and 1993 to 2001.
* The Treasury rates are daily time series going from 1990 to 2001. The yields on Treasury securities at ‘constant maturity’ are interpolated by the U.S. Treasury from the daily yield curve. These market yields are calculated from composites of quotations obtained by the FD Bank of New York. The constant maturity yield values are read from the yield curve at fixed maturities, currently 3 and 6 months and 1, 2, 3, 5, 7, 10, and 30 years.
* The Eurodollar interbank interest rates are bid rates with different maturity dates they are daily data in range 1990- 1996.

To perform the analysis in the methodology is the *generalized hurst exponent* introduced in *Raffaello Morales, T. Di Matteo, Ruggero Gramatica, Tomaso Aste, Dynamical generalized Hurst exponent as a tool to monitor unstable periods in financial time series,Physica A: Statistical Mechanics and its Applications,.* This approach gives the opportunity to combine sensitivity to dependance of data.

GHE is a method to exploit the scaling properties of the data thought the qth-order moments of the distribution of the increments and it is associated with the long-term statistical dependence of time series S(t), with t = (1, 2, . . . , k, . . . , 1t ), defined over a time-window 1t with unitary time-steps.

Kq(τ) =

where τ can vary between 1 and τmax and ⟨·⟩ denotes the sample average over the time-window. Note that for q = 2, Kq (τ) is proportional to the autocorrelation function: C(t,τ) = ⟨S(t + τ)S(t)⟩. The generalized Hurst exponent is then defined from the scaling behavior of Kq (τ) when the following relation holds:

Kq(τ) ∝ τqH(q). 

In this process with scaling behavior can be of two classes:

1. Processes with H(q) = H, i.e. independent of q. Uniscaling (or unifractal) processes and their scaling behavior is uniquely determined by the constant H (Hurst exponent or self-affine index)
2. Processes with H(q) not constant. Multiscaling (or multifractal) processes and each moment scales with a different exponent.

The application to different markets condition of the methodology lead to some interesting conclusions in particular that the scaling behaviors are almost universal in financial markets.

Using q-moment to investigate they shown that Hurst exponent is a powerful instrument to characterize the structure of scaling properties.

The novelty of this study lies in its empirical analysis of various stock indices, which demonstrates the sensitivity of the exponent H(2) to the level of market development. The analysis covers a wide range of indices, including the Nasdaq 100, S&P500, Nikkei 225, Dow Jones Industrial Average, CAC 40, and All Ordinaries index, all exhibiting H(2) values below 0.5. On the other hand, indices like the Russian AK&M, Indonesian JSXC, and Peruvian LSEG exhibit H(2) values above 0.5.

Another conclusion is that emerging patterns in the scaling behaviors of interest rates and exchange rates that are connected to specific market conditions. For instance, there is a notable deviation in the scaling exponent for the 3-month maturity, which strongly correlates with central bank decisions. This implies that the scaling exponents can be used as a simple and effective empirical measure for characterizing the development of financial markets, surpassing the capabilities of traditional risk control measures such as standard deviation or Sharpe Ratio.

About the power spectra the study demonstrates that the generalized Hurst exponent method accurately describes the scaling behavior even in the frequency domain.

All this methodology and knowledge from the application in the field of market efference are the basis of the methodology adopted in this work and illustrated in the next chapter.

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